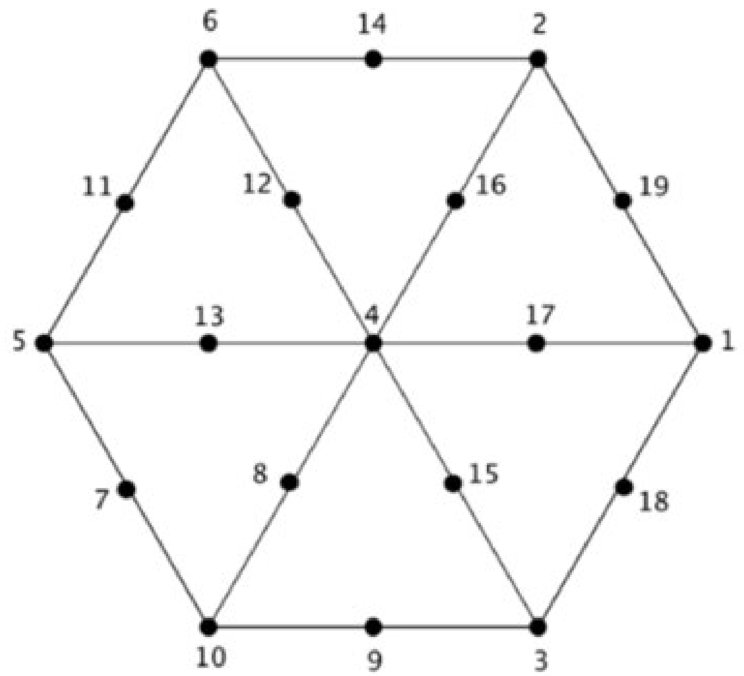
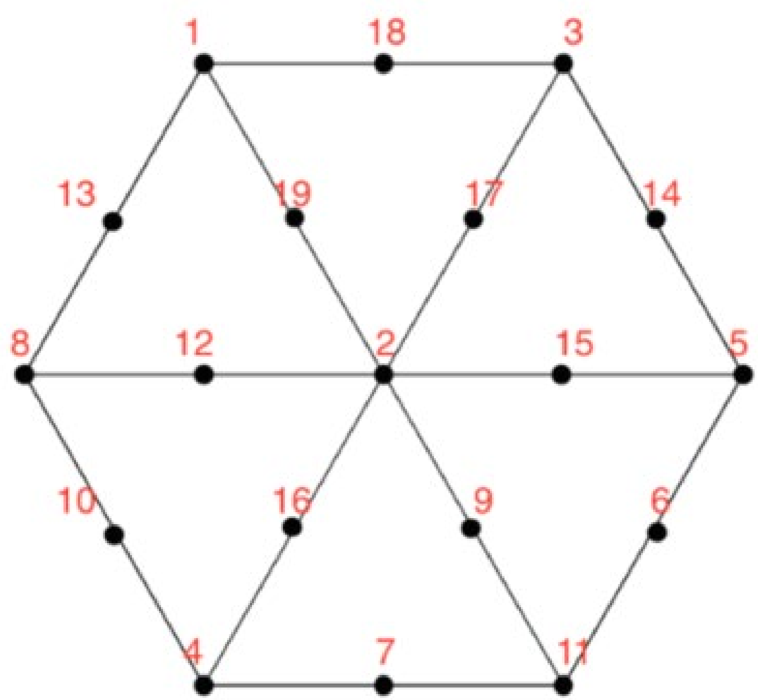
19 Dots

Here are 19 dots arranged in a hexagon. Your task is to label the dots with the numbers 1 to 19 so that each set of three dots that lie along a straight-line segment add up to 22.

Solution:

Here are two solutions. There may be others.

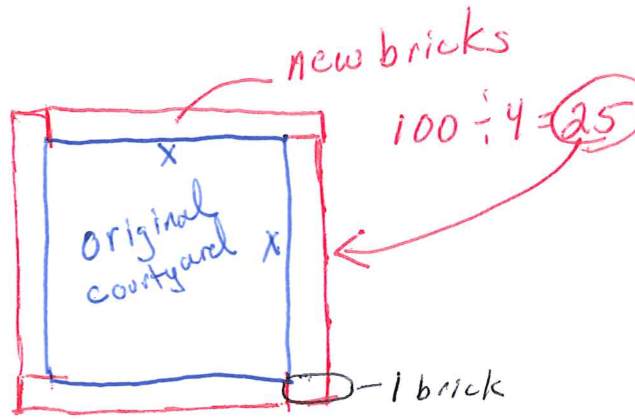




And 100 More

Liam the Mischievous
Leprechaun

A man completely paved a square courtyard with square tiles. Liam arrived one night and decided the courtyard was too small. He dug up the courtyard, removed all the tiles, stacked them neatly, and added 100 more to the stack. The next morning the man returned to the courtyard to find a stack of tiles with a note instructing him to build another square courtyard using all of the tiles in the stack.



$x = 25 - 1 = 24$

new dimension = 24 + 2 = 26

or 25 + 1 = 26

Your task is to determine how many tiles were used in the second courtyard. All the tiles were the same size. Be prepared to justify your solution.

New courtyard $26 \times 26 = 676$

original courtyard $24 \times 24 = 576$

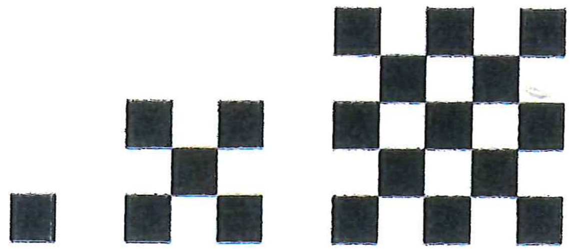
When you have solved this problem, go to Room 1069 to present your solution to the Master Teacher.

$676 - 576 = 100$

Checkerboard Pattern

Your task is to determine an explicit formula for the number of tiles in the n^{th} step for the following pattern.

The tiles are just the black squares.



n = the step #

1st: 1 tile; 2nd: 5 tiles; 3rd: 13 tiles; 4th: 25 tiles

T = # of tiles

Be prepared to justify each aspect of your formula with respect to the pictorial representation.

Before going to the Master Teacher, decide whether you are trying for:

1 Puzzle Piece (formula without pictorial representation)

$T = 2n^2 - 2n + 1$

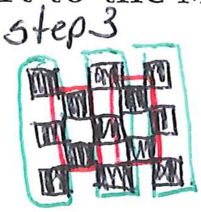
Or

4 Puzzle Pieces (formula with complete pictorial representation)

Once you report to the Master Teacher, you are locked into your decision.

This is one way

step 3



$3 \times 3 \rightarrow n \times n = n^2$

$2 \times 2 \rightarrow (n-1)(n-1) = n^2 - 2n + 1$

add together

$n^2 + n^2 - 2n + 1$

$2n^2 - 2n + 1$

When you have solved this problem, go to Room 1021 to present your solution to the Master Teacher.

Coin Game

Sparkles O'Looney and Blarney O'Stone each has a set of gold coins. They decided to play four rounds of a game where, after each round, the loser must give the winner as many gold coins as the winner has at that time. Both leprechauns ended up with 16 gold coins. If Sparkles won the first two rounds and Blarney won the last two rounds, how many gold coins did each one have at the start of the game?

Your task is to determine how many gold coins Sparkles had and how many gold coins Blarney had when the game began.

When you have solved this task, go to Room 1065 to present your solution to the Master Teacher.

One Way: Work Backwards

	After round 4	After round 3	After round 2	After round 1	Start
Sparkles	16	24	28	14	7
Blarney	16	8	4	18	25

Growing Squares

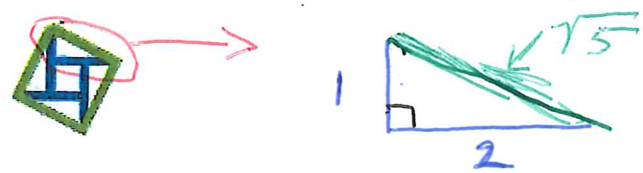
Molly McDoodle was doodling the other day and drew a little square like this: 

Area = 1 sq unit Perimeter = 4 units

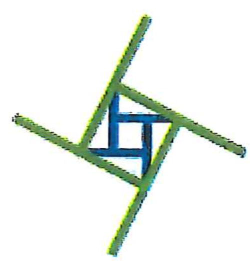
And she supposed that the side was one (something) long. Well, she wondered what would happen if she drew the four lines a bit longer, in fact twice as long so that the extra bits stuck out.

This was quite a nice little design, she thought, and then she noticed that it looked as though the ends of these lines could make a square. So, she drew one! She used a different color to show this new square.

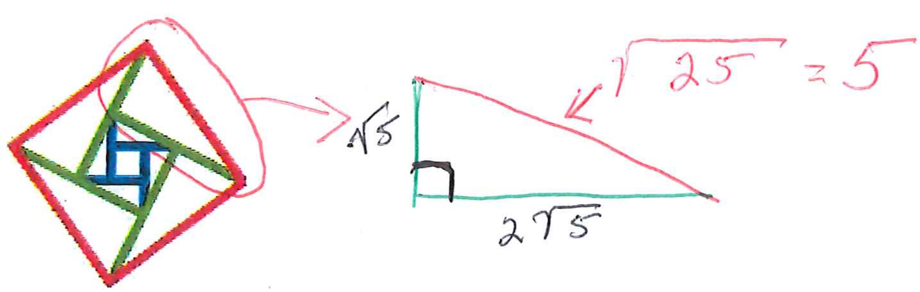
Use pythagorean Theorem



Now, mathematical patterns usually go on repeating themselves so she used that idea to pretend that this new green square was her first one and so she drew the extra bits again, so that the lines were twice as long as the square. Her new shape looked like this:

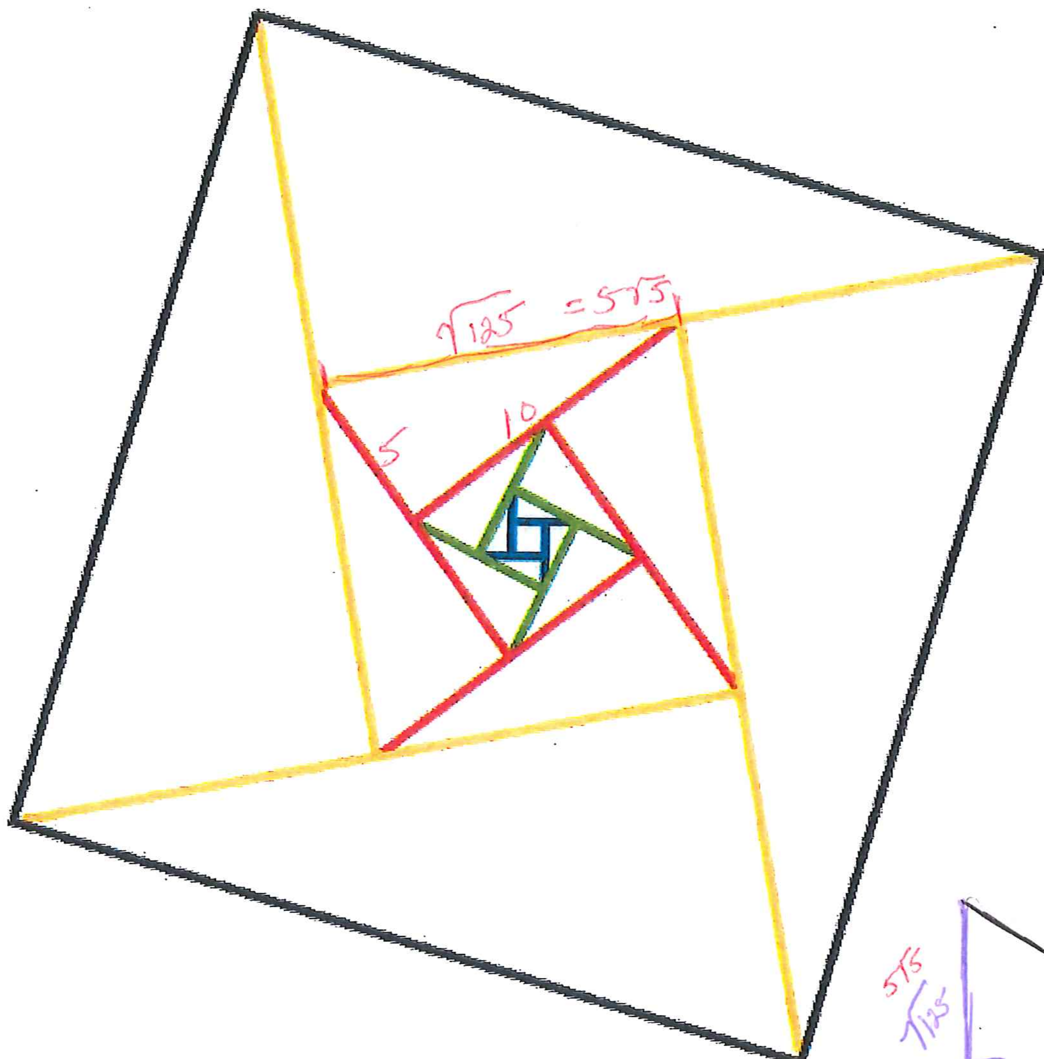


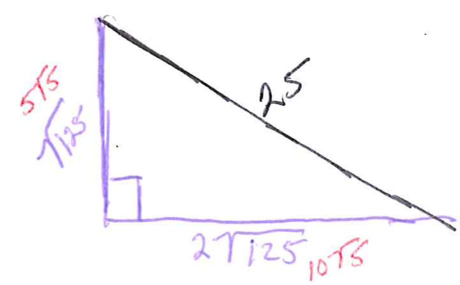
So she went on! A new square appeared, now red!



(continued on back)

She really liked what was happening here. She continued with this mathematical pattern until creating this final drawing.





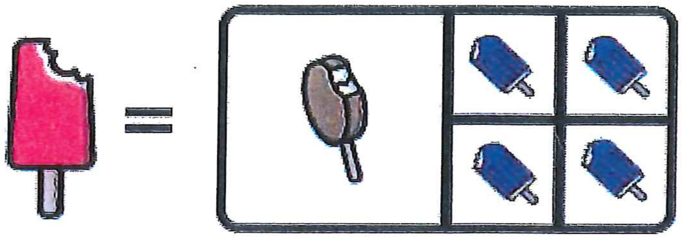
Your task is to determine the perimeter and area of the black square if the area of the blue square is one square unit.

$A = 25^2 = 625$ sq units $P = 100$ units

When you have solved this task, go to Room 1016 to present your solution to the Master Teacher.

Ice Cream Challenge

Your task is to determine the value of each frozen treat. Be prepared to justify your solution.

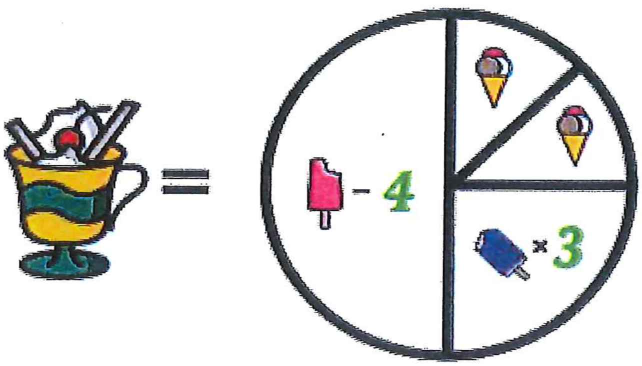


Area Model

4B = 1E

2E = 1R

8B = 1R



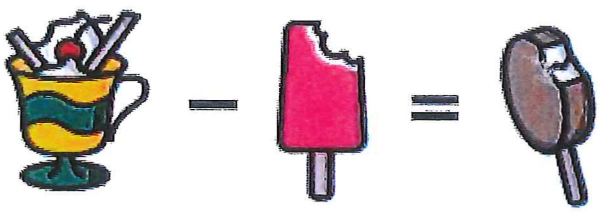
2C = 3B

2C + 3B = R − 4

6B = R − 4

P = 6B + (8B − 4)

P = 14B − 4



(14B − 4) − (8B) = 4B

6B − 4 = 4B

2B = 4

B = 2

E + C = 11

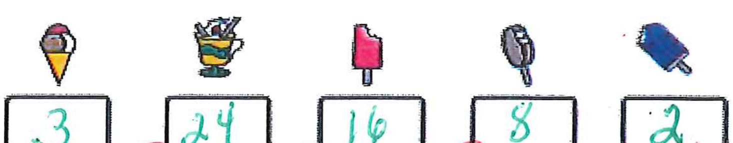
[illegible]

[illegible]

4B + C = 11

4(2) + C = 11

C = 3

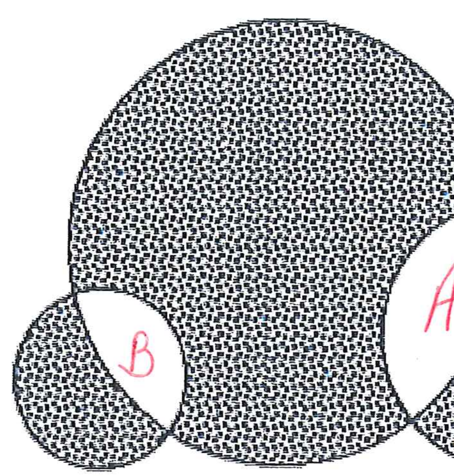


C for cone, P for Parfeit, R for Red popsicle, E for Eskimo pie, B for Blue popcicle

When you have solved this problem, go to Room 1019 to present your solution to the Master Teacher.

There are many ways to explain this.

Inside And Out



The circles above have radii 12 units, 8 units and 5 units.

Your task is to determine the difference between the shaded region inside the big circle and the shaded region outside the big circle.

Area of largest circle $= 144\pi$

Area of (midsize) circle $= 64\pi$

Area of smallest circle $= 25\pi$

marked above in Red

Area inside the largest circle $= 144\pi - (A+B)$

Area outside the largest circle $= (64\pi - A) + (25\pi - B)$

Difference $= [144\pi - (A+B)] - [(64\pi - A) + (25\pi - B)]$

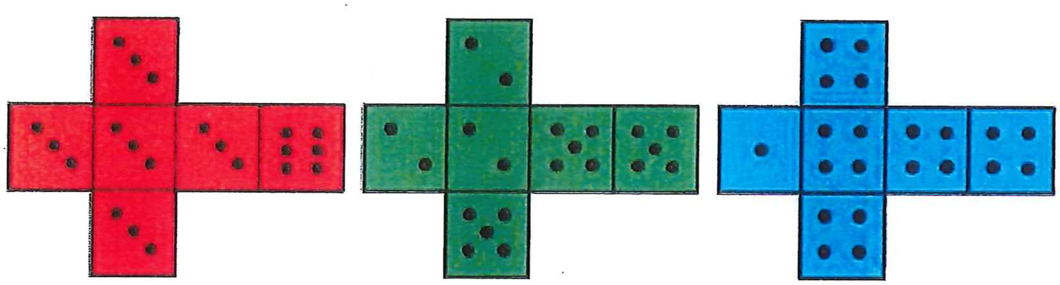
$= 144\pi - A - B - 64\pi + A - 25\pi + B = 55\pi$

When you have solved this problem, go to Room 1056 to present your solution to the Master Teacher.

Lucky Dice

Two people are playing a game using the special colored dice shown below. There is one red die, one green die, and one blue die. To play the game, each person picks one die.

- Each player rolls their die and the person who rolls the highest number wins the round.
- The players roll their dice 30 times and keep track of who wins each round.
- Whoever has won the most rounds after 30 rolls wins the game.



Your task is not to play the game, you should:

A. Pick any two colors and tell which color would most likely win and give the probability of that color winning.

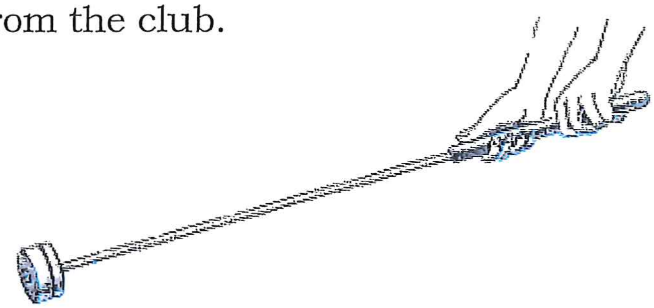
P(Red winning over Green) = 21/36 P(Blue winning over Red) = 25/36 P(Green winning over Blue) = 21/36

B. If you were playing the game, is it better to choose your color first or second? Explain. Go second, no matter what color your opponent chooses there is always a die on the table that has a better chance of winning which you can choose.

When you have solved this problem, go to Room 1010 to present your solution to the Master Teacher.

Mulligan's Club

Mulligan's Company wants to manufacture a practice golf club that will have a variable amount of weight at the end of the club. The golfer will begin by placing a1-ounce weight on the end of the club and then build up gradually in 1-ounce steps to reach a maximum of 15 ounces. Weights can be added and removed from the club.



Séamus Power suggests that it will be most convenient to have as small a number of individual weights as possible. Mulligan's Company likes this idea, but they want to know the specific value of each of the weights Séamus Power suggests.

Your task is to determine the smallest number of individual weights and their values that Séamus can have available for a session with his practice club to have any weight from one to fifteen ounces in one-ounce increments. (Ignore the weight of the club itself.)

Make the weights in powers-of-2

1oz, 2oz, 4oz, 8oz. [4 weights]

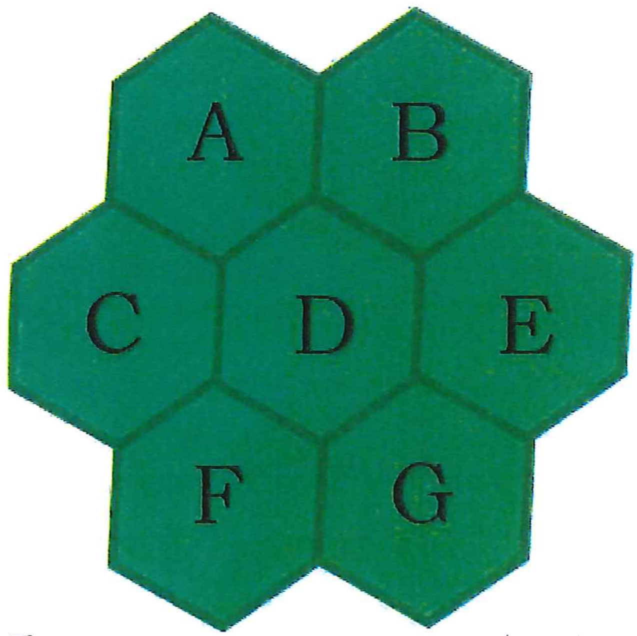
1 → 1oz
2 → 2oz
3 → 2+1
4 → 4
5 → 4+1
6 → 4+2
7 → 4+2+1
8 → 8
9 → 8+1
10 → 8+2
11 → 8+2+1
12 → 8+4
13 → 8+4+1
14 → 8+4+2
15 → 8+4+2+1

When you have solved this task, go to Room 1013 to present your solution to the Master Teacher.

Patchy Situation

The Leprechaun Quilting Society is preparing a patchwork quilt to be auctioned off to raise money for the Math League. Just as the members themselves are diverse, the quilt is a colorful medley of fabrics and patterns. A section of the finished quilt is shown below. Each piece is made of a different material in a different pattern and in a different basic color. From the information given, determine, for each piece (A through G in the illustration), its basic color, pattern (one pattern is pentagrams), and fabric (one section is made out of wool).

1. The zigzag patch is predominantly yellow. It is adjacent to patch D, the pink patch, and the green dotted patch. Patch G is satin.
2. The navy patch is adjacent to the geometric patch (which is not the polyester one), the one with a π pattern, and patch D.
3. The parallel line patch (which isn't patch E) is in a direct line with the brown patch, with only patch D between them. The brown patch is adjacent to the nylon patch.
4. Patch A is red, but it isn't the velvet one with the π pattern. Patch C is checked, but isn't the pink, silk one.
5. The orange patch is made of cotton.



Your task is to the color, design, and fabric for each of the hexagonal quilt pieces. List these by the hexagonal quilt pieces' letters.

A - red, geometric, wool

B - pink, parallel lines, silk

C - navy, checked, nylon

D - orange, pentagrams, cotton

E - yellow, zigzag, polyester

F - Brown, π, velvet

G - green, dotted, satin

When you have solved this task, go to Room 1009 to present your solution to the Master Teacher.

Relationships

Sean and Conner, the playful Leprechauns, have found a box of golden Relational GeoSolids and a bag of rice. Since they are always drawn to gold, they started playing with them. They discovered that when they filled them, there were relationships between the solids. In their wonderful math class, taught by Lady MacMath, they had learned a formula for the area of a circle. They noticed that some of the shapes had involved circles. From this, they developed a way to find the volume of a cylinder.

Given: Area of circle is πr^2

Volume of cylinder is a circle base times height (In relational solids $h = 2r$)

The overachieving leprechauns continued playing with the solids and found more relationships until they developed a formula for finding the volume of the sphere. $V = \frac{4}{3}\pi r^3$

$V = \pi r^2 h$

$V = \pi r^2 \cdot 2r$

$V = 2\pi r^3$

Your task is to be as clever as the leprechauns and, using the relationships, you notice in the GeoSolids, demonstrate to the Master Teacher how to find the volume of a sphere, and state the formula. The Master Teacher will expect to see your algebraic reasoning.

Through investigation with rice & solids

3 cones = 1 cylinder

or

1 cone = $\frac{1}{3}$ cylinder

and 1 cone = 1 hemisphere

1 sphere = 2 hemispheres

When you have solved this task, go to Room 1062 to present your solution to the Master Teacher.

Volume 1 cylinder = $2\pi r^3$

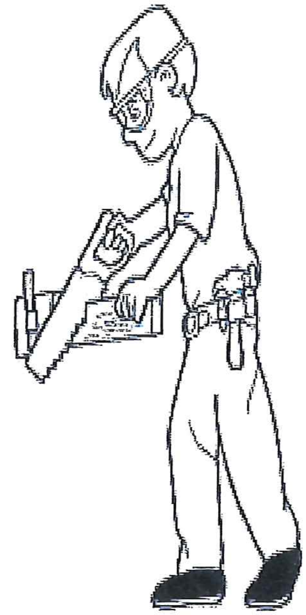
1 sphere = 2 cones

so Volume 1 cone $= \frac{1}{3}(2\pi r^3)$

Volume 1 sphere $= 2(\frac{1}{3})(2\pi r^3)$

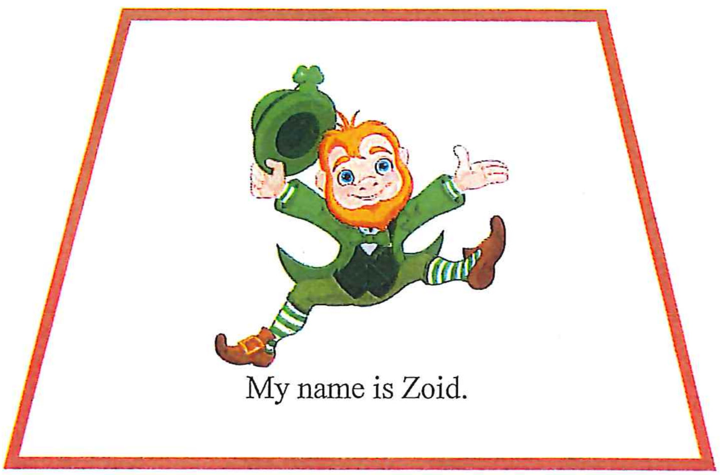
Volume 1 sphere $= (4/3)\pi r^3$

The Long and Short of It



Ryan O'Toole is building a trapezoidal leprechaun trap. He has a wooden board measuring (y + 8) feet in length to create the bases for the leprechaun trap. The board is cut into two pieces in the ratio 3:7.

short base — long base



set up ratio

$\frac{\text{short base}}{\text{whole board}}$

$\frac{3}{10} = \frac{\text{short base}}{y+8}$

$10(\text{short base}) = 3(y+8)$

$\text{short base} = \frac{3(y+8)}{10}$

Your task is to determine the length of the board for the shortest base.

When you have solved this task, go to Room 1070 to present your solution to the Master Teacher.

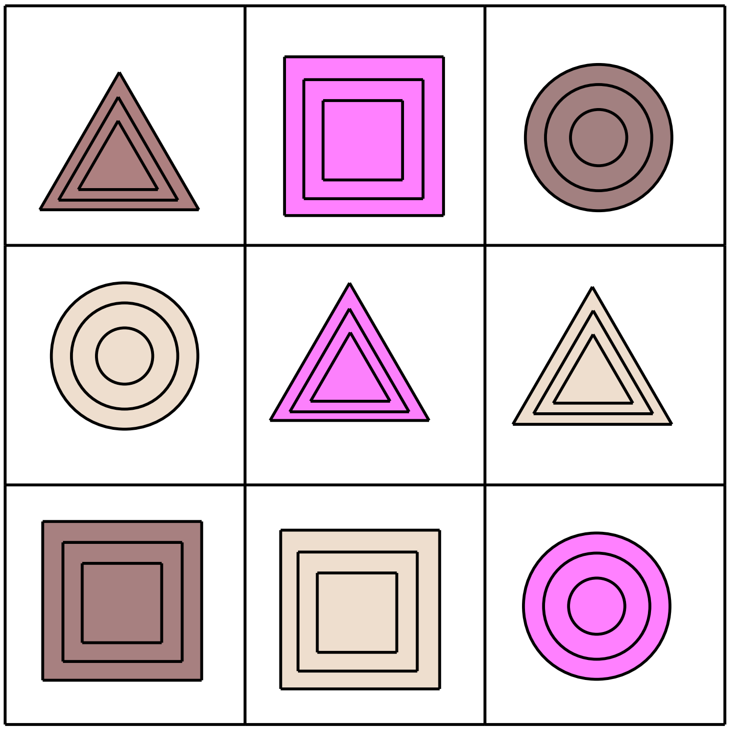
Building Blocks Answer Key

Step	Work	Number of Squares
1	$3*1 = 3*2^0$	3
2	$3*2 = 3*2^1$	6
3	$3*4 = 3*2^2$	12
4	$3*8 = 3*2^3$	24
n		$3*2^{n-1}$

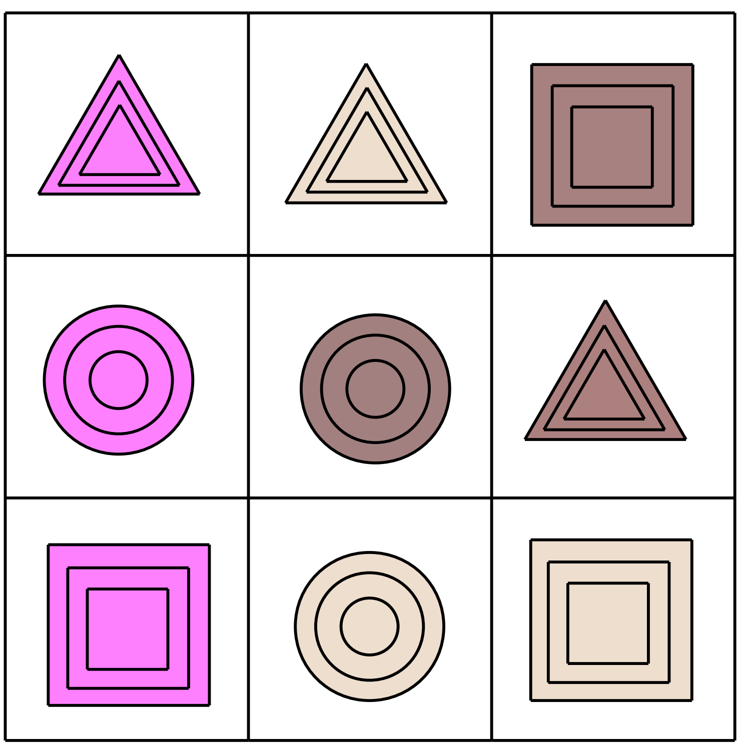
Explanation:
The 3 is the height of each step. The 2 is the scale factor from step to step.

Chocolate Fix Solution

Advanced



Expert



Coordinate Parallelograms

Directions: Fill in the x and y coordinates using whole numbers 1 – 9, without repetition, so that the points make a parallelogram.

(☐ , ☐) (☐ , ☐)

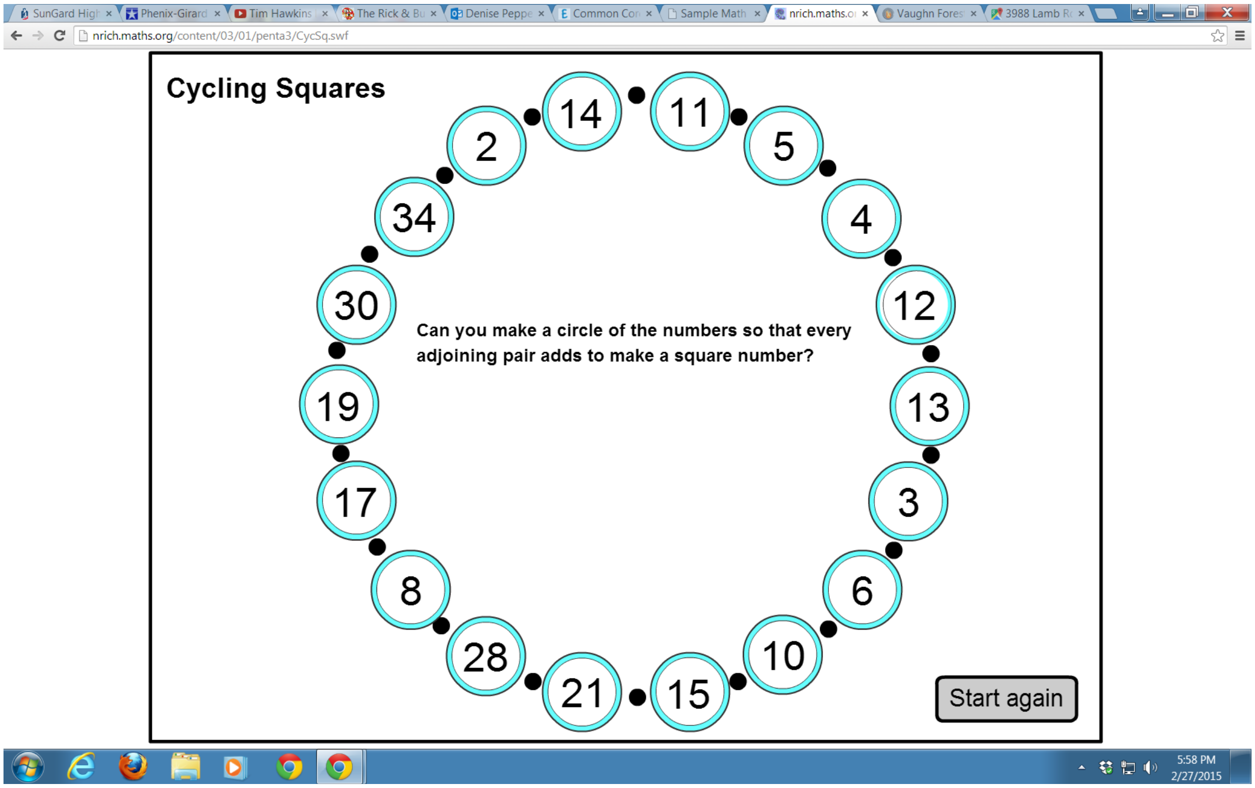
(☐ , ☐) (☐ , ☐)

Solution:

There are multiple answers:

- (1,7) (6,5) (3,4) (8,2)
- (posted by Kevin Rees) I only found eight possible solutions, but I'm willing to be wrong.
  All are transformations of the first solution given – reflect it over the midline (y = 4.5) for another, over the other midline (x = 4.5) and across the line y = x. This gives you four solutions that all have rotational symmetry with each other about a circle centered at (4.5, 4.5). To get the other four solutions translate the first four solutions by the vector.

Answer Key to Cycling Squares



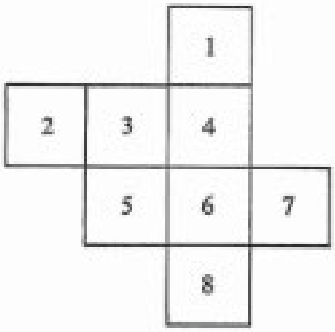
Four Brothers Answer

Alfred & Horatio are lying.
Indigo & Bernard are telling the truth.

Mathematical Constants (4 points, -1 for hint)

Problem:

Eight numbered cards lie face down on a table in the relative positions shown in the diagram below.



Of the eight cards:

1. Every Ace borders on a King.
2. Every King borders on a Queen.
3. Every Queen borders on a Jack.
4. No Queen borders on an Ace.
5. No two cards of the same kind border on each other.
6. There are two Aces, two Kings, two Queens, and two Jacks.

Which kind of card—Ace, King, Queen, or Jack—is card number six?

Use constants

Ace π pi

King ϕ phi

Queen τ tau

Jack e

The hint will be what is in position 6.

Solution:

Suppose card number six were an Ace. (a) Then neither cards number seven nor eight could be an Ace, from [5]; could be a Queen, from [4]; could be a King, from [2]. (b) Then at most one card of cards number seven and eight could be a Jack, from [3]. So, from [6], card number six is not an Ace.

Suppose card number six were a Queen. (a) Then none of cards number four, five, seven, and eight could be a Queen, from [5]; or could be an Ace, from [4]. (b) Then, from [6], two Aces and a Queen would be cards number one, two, and three; which, from [4] and [5], is impossible. So, from [6], card number six is not a Queen.

Suppose card number six were a Jack. (a) Then neither cards number seven nor eight could be an Ace, from [1]; could be a Jack, from [5]; could be a King, from [2]. (b) Then at most one card of cards number seven and eight could be a Queen, from [2]. So, from [6], card number six is not a Jack.

Then card number six is a King.

It is possible to determine cards number one through six. Since card number six is a King, card number five or four is a Queen, from [2] and [3]. If card number five is a Queen, then card number three is a Jack, from [3]. Then card number two is not a Queen, from [2], and cards number one and four are a King and a Queen, respectively, from [2]. Then, from [6], card number two must be a Jack, which contradicts [5]. So card number five is not a Queen and card number four is a Queen. Then neither of cards number one and three is a Queen, from [5]; neither of cards number seven and eight is a Queen, from [3]; and card number five is not a Queen, from previous reasoning. So card number two is a Queen. Then card number three is a Jack, from [3]; card number one is a King, from [2]. Then card number five is an Ace, from [5] and [6]. A Jack and an Ace are left for cards number seven and eight.

```
solving Pentominoes3x20:

solution 1:
01,1 02,0 02,1 02,2 03,1 X
00,0 00,1 00,2 01,0 01,2 U
03,2 04,1 04,2 05,1 05,2 P
03,0 04,0 05,0 06,0 07,0 I
06,1 06,2 07,2 08,2 09,2 L
07,1 08,0 08,1 09,0 10,0 N
09,1 10,1 10,2 11,0 11,1 F
11,2 12,0 12,1 12,2 13,2 T
13,0 13,1 14,1 14,2 15,2 W
14,0 15,0 15,1 16,0 17,0 Y
16,1 16,2 17,1 18,0 18,1 Z
17,2 18,2 19,0 19,1 19,2 V

U U X P P P L L L L F T T T W W Z V V V
U X X X P P L N N F F F T W W Y Z Z Z V
U U X I I I I I N N N F T W Y Y Y Y Z V

solution 2:
01,1 02,0 02,1 02,2 03,1 X
00,0 00,1 00,2 01,0 01,2 U
03,2 04,1 04,2 05,1 05,2 P
03,0 04,0 05,0 06,0 07,0 I
06,1 06,2 07,1 08,0 08,1 Z
07,2 08,2 09,1 09,2 10,2 Y
09,0 10,0 10,1 11,1 11,2 W
11,0 12,0 12,1 12,2 13,0 T
13,1 13,2 14,0 14,1 15,1 F
14,2 15,2 16,1 16,2 17,1 N
15,0 16,0 17,0 18,0 18,1 L
17,2 18,2 19,0 19,1 19,2 V

U U X P P P Z Y Y Y Y W T F N N N V V V
U X X X P P Z Z Z Y W W T F F F N N L V
U U X I I I I I Z W W T T T F L L L L V
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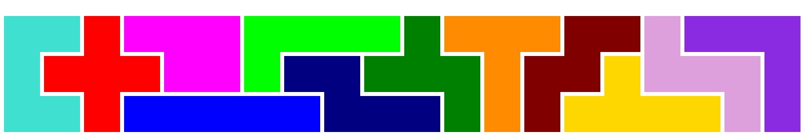
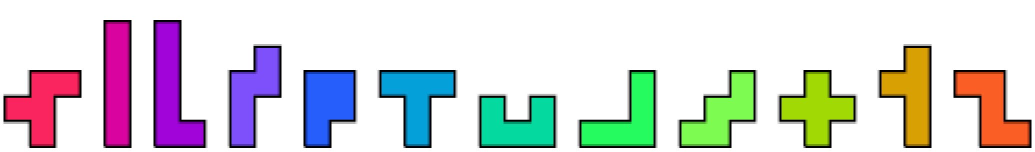
Pentominoes

Rectangles

- 3x20: 2 solutions





F I L N P T U V W X Y Z

Pick a House Solution

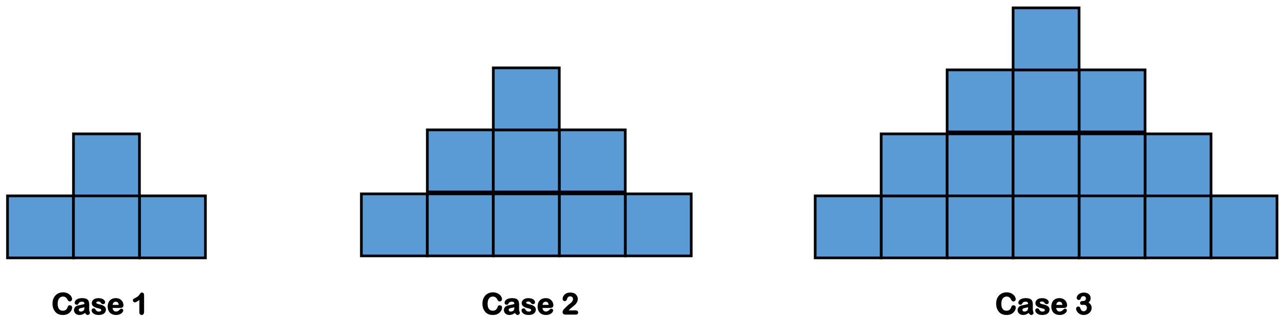
As you are walking down Main Street in a small town, you notice three pretty houses next to one another. In each house lives two children. One house has a boy and a girl, one house has two girls, and the last house has two boys. If you were to walk into a house at random and see a girl, what is the probability that the other child in the house is also a girl?

Answer: 2/3 or 0.666 or about 67%.

Solution: There are three girls and each is equally likely to be the girl in a house with another girl. For example, if Jane and Susan live in one house, Carly and Tom in another, and Mike and David in the third then: if you see Jane you have a chance to see a girl, if you see Susan you have a chance to see a girl, if you see Carly, you do not have a chance to see a girl. That is three chances to see a girl first and only two possibilities to see a girl second

Growing Shapes

This is a two-part problem.



Part One:

Without any numbers or algebra, describe how you see the shapes growing. Once you have one description, look again to find another way. Report both of your descriptions to the Master Teacher. When the Master Teacher is satisfied with your descriptions, you will receive part two of this problem.

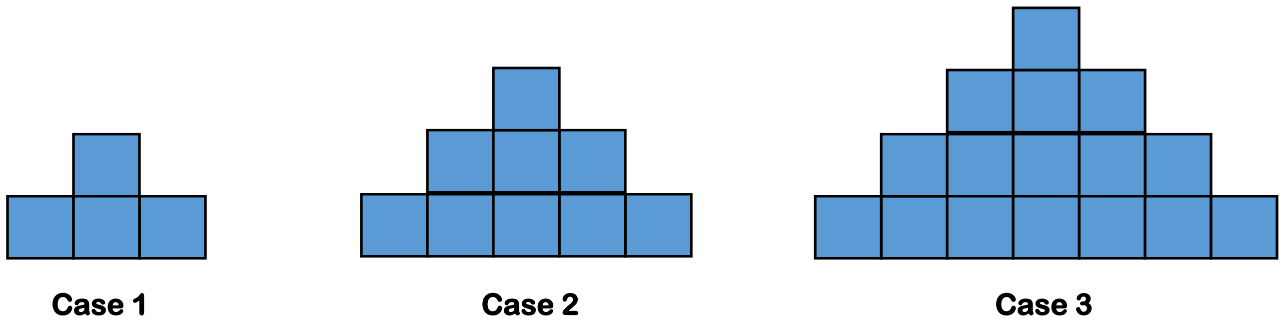
Solution:

Answers will vary. This problem is designed to be a math talk. For sample solutions, go to Jo Boaler's TEDxStanford talk, "How you can be good at math and other surprising facts about learning.

https://www.youtube.com/watch?v=3icoSeGqQtY

Growing Shapes

This is a two-part problem.



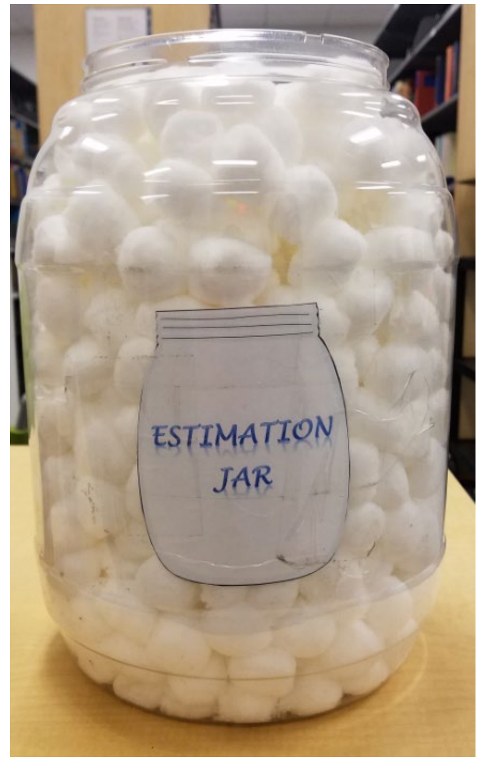
Part Two:
Your task is to generate a formula that will determine the total number of tiles necessary to build the n^{th} case for the growing shapes. Be prepared to justify each aspect of your formula with respect to the growth of the shapes or the structure of the geometric aspects of the shapes. You will earn 1 puzzle piece for the formula and an additional 2 puzzle pieces for the explanation.

Solution:

$(n + 1)^2$

Quality of explanation will decide number of points received.

How Many?



Using your best estimation skills and tools at your disposal estimate, determine how many cotton balls are in this container.

Tools that you may use are in your tub include, but are not limited to:

- a cotton ball.
- a yogurt container.
- a medium size plastic container.
- a measuring tape.

You may come to Historic Room 1056 to investigate the container filled with ***loosely, packed*** cotton balls.

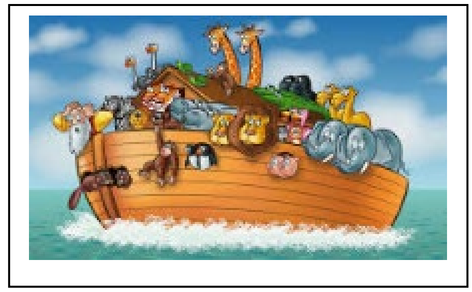
You are not receiving puzzle pieces solely based on the estimation value. Instead you will receive one or two puzzle pieces based on the MATH THINKING behind your estimation.

When you have done the math, and you are ready to justify your estimate, return to the Master Teacher.

Solution:

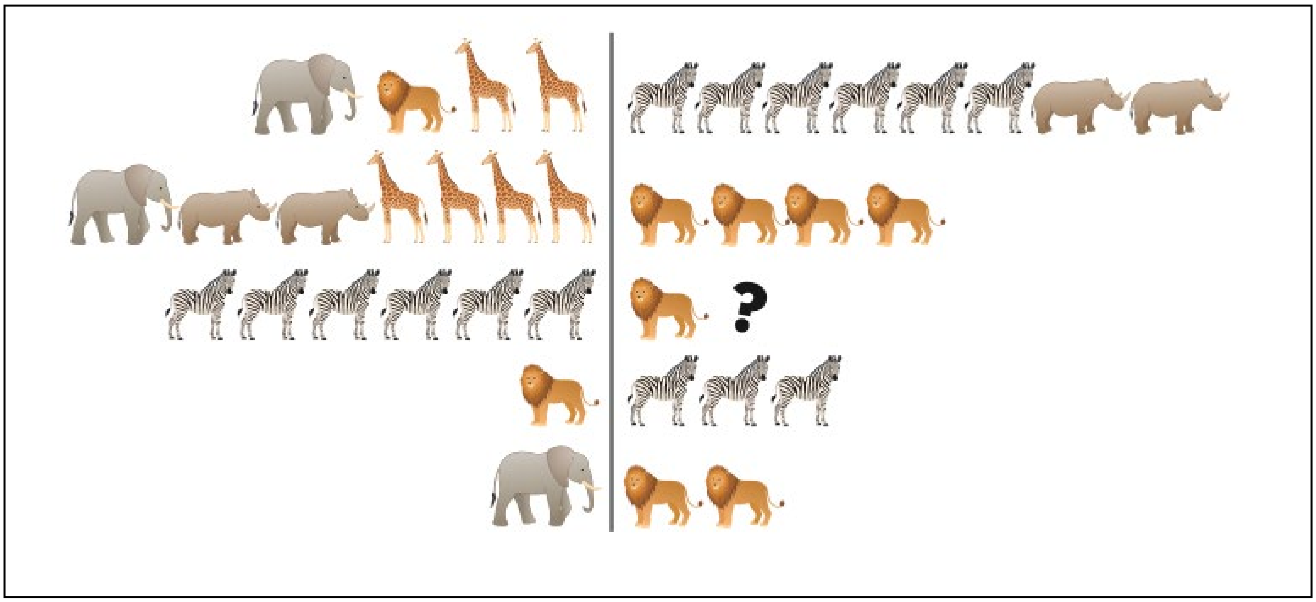
There are 620 cotton balls in the container. Explanation should include proportional reasoning based on the benchmark containers provided to the students. By our calculations there were 11 cotton balls per yogurt container.

Noah's Ark



SOLUTION

Noah wants his Ark to sail along on an even keel. The Ark is divided down the middle, and on each deck the animals on the left exactly balance those on the right-- all but the third deck.



Your task is to determine how many giraffes are needed in place of the question mark. Be prepared to justify your answer.

Six giraffes

NUTS

SOLUTION

George Crackham put five bags on the table. On being asked what they contained, he said:

"Well, I have put one hundred nuts in these five bags. In the first and second there are altogether fifty-two nuts; in the second and third there are forty-three; in the third and fourth, thirty-four; in the fourth and fifth, thirty."

Your task is to determine how many nuts are in each bag. Be prepared to explain your problem solving method and justify your answer.

Bag 1 = 27

Bag 2 = 25

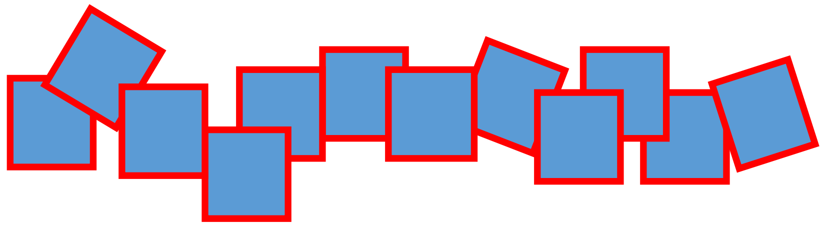
Bag 3 = 18

Bag 4 = 16

Bag 5 = 14

Open Box Challenge

This is a two-part problem.



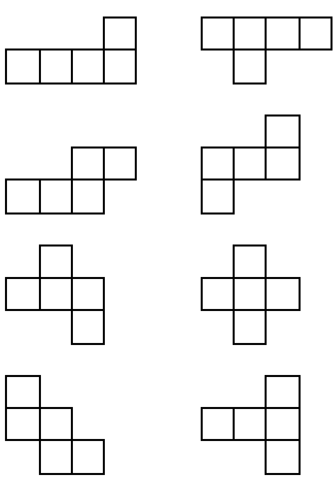
Part One:

Using the 1" paper squares, create nets that will fold into a 1" x 1" x 1" open box. Each net MUST BE UNIQUE. Tape the squares together to form each unique net.

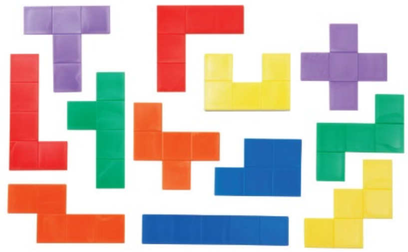
Once you think you have created all the possible unique nets, take them to the master teacher. This is a ONE TIME OPPORTUNITY. You will not be allowed to submit this answer more than one time.

When you present Part One's answer, you will receive Part Two from the Master Teacher.

Solution nets



Open Box Challenge

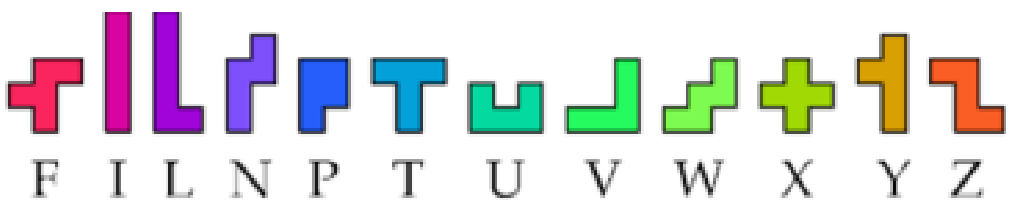


Part Two

The Master Teacher has given you a set of pentominoes, a practice sheet, and a 3" x 20" rectangular template. Your task is to fill the rectangle with all 12 pentominoes. You must stay within the rectangle's boundaries with no gaps and no overlaps.

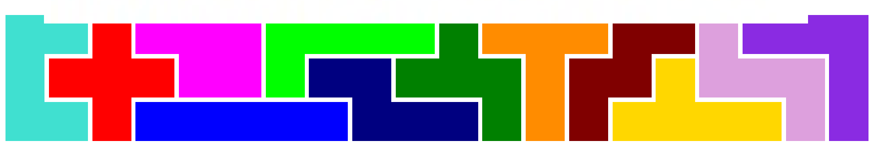
Before coming to the Master Teacher, be prepared to explain how you derived your solution.

Once you have filled the rectangle, CAREFULLY bring your filled rectangular template to the Master Teacher. Remember that there is a white board in your room to help you with this.



Solution1:

U	U	X	P	P	P	L	L	L	L	F	T	T	T	W	W	Z	V	V	V
U	X	X	X	P	P	L	N	N	F	F	F	T	W	W	Y	Z	Z	Z	V
U	U	X	I	I	I	I	I	N	N	N	F	T	W	Y	Y	Y	Y	Z	V



Solution 2:

U	U	X	P	P	P	Z	Y	Y	Y	Y	W	T	F	N	N	N	V	V	V
U	X	X	X	P	P	Z	Z	Z	Y	W	W	T	F	F	F	N	N	L	V
U	U	X	I	I	I	I	I	Z	W	W	T	T	T	F	L	L	L	L	V

Powers of

SOLUTION

Today is March 9, 2019 ($3^1/3^2/2019$) . This problem is to pay respect to the day.

$$3^x + 3^{x-1} + 3^{x-2} = 351$$

Your task is to solve for x.

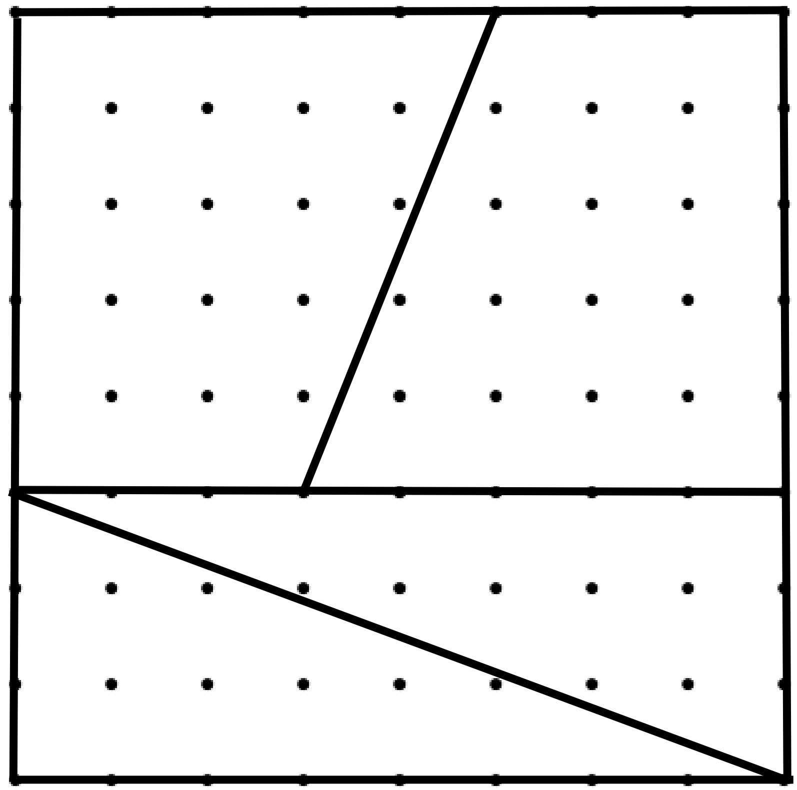
Be prepared to justify your solution.

$$3^{x-2}(3^2 + 3^1 + 1) = 351$$

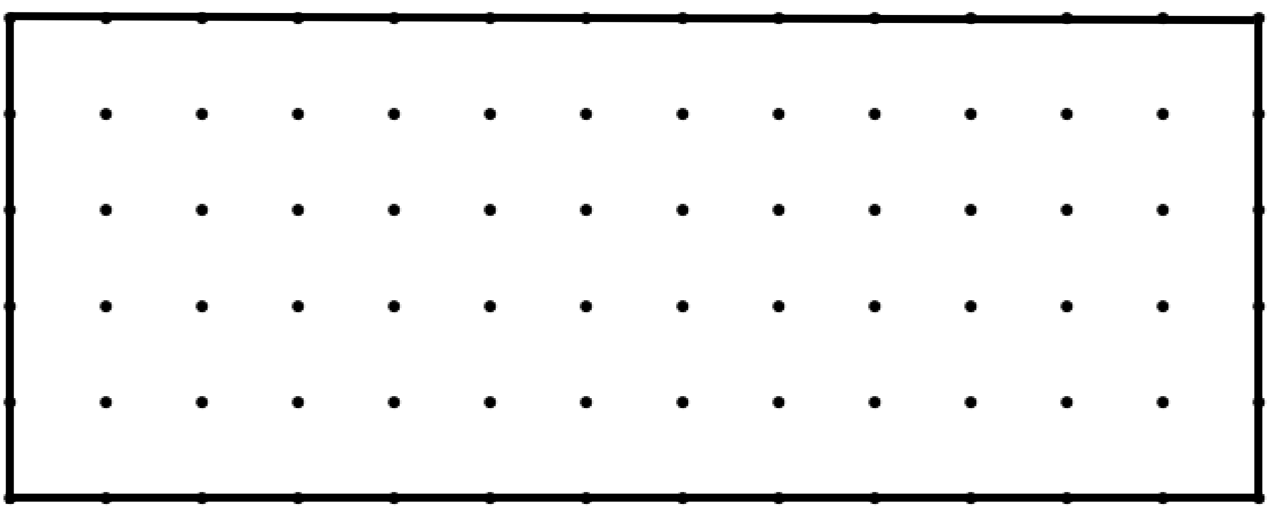
X = 5

Square to Rectangle – Part 1

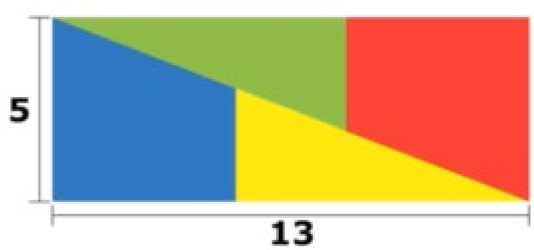
Take an 8 by 8 square, and cut it up as shown to the right.



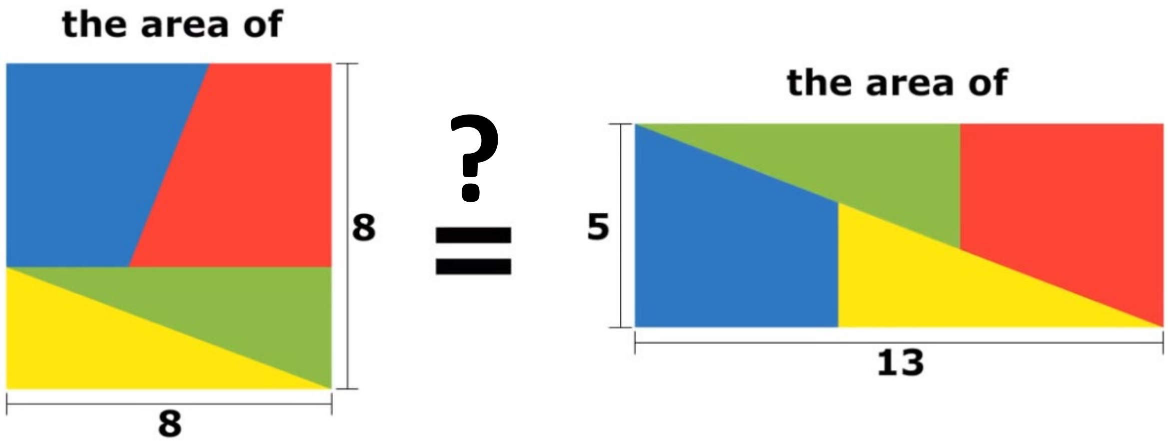
In the rectangle below, rearrange the pieces to make a 5 by 13 rectangle.



Solution:



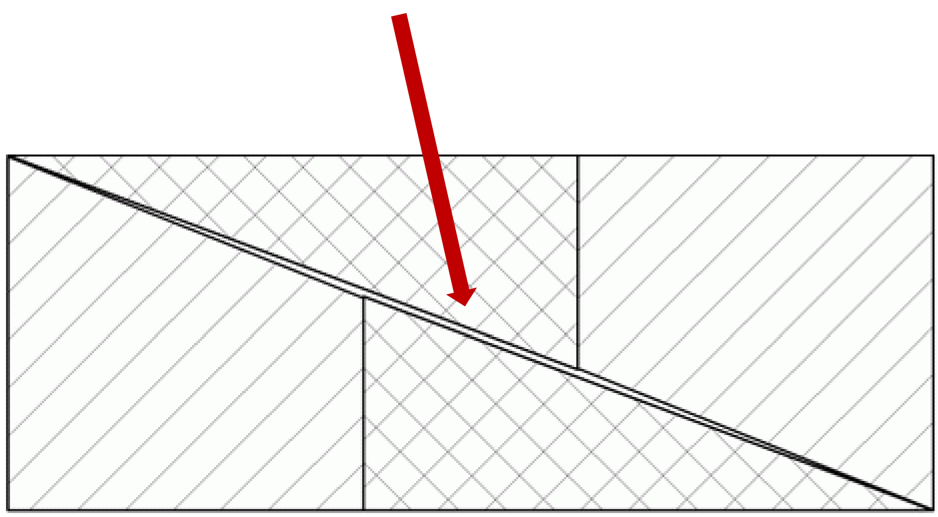
Square to Rectangle: Part 2



It seems we can use the same pieces to make two shapes with different areas! Or can we? Can you explain why or why not this can or cannot be done? Be very detailed in justifying your answer.

Solution:

What looks to be a diagonal in the 5 by 13 rectangle is not a straight line. The slopes for the triangles is 3/8 and the slope on the trapezoids is 2/5, thus creating a concave image that appears to be straight. The extra square unit of area is in this “slice”.



All the Pets

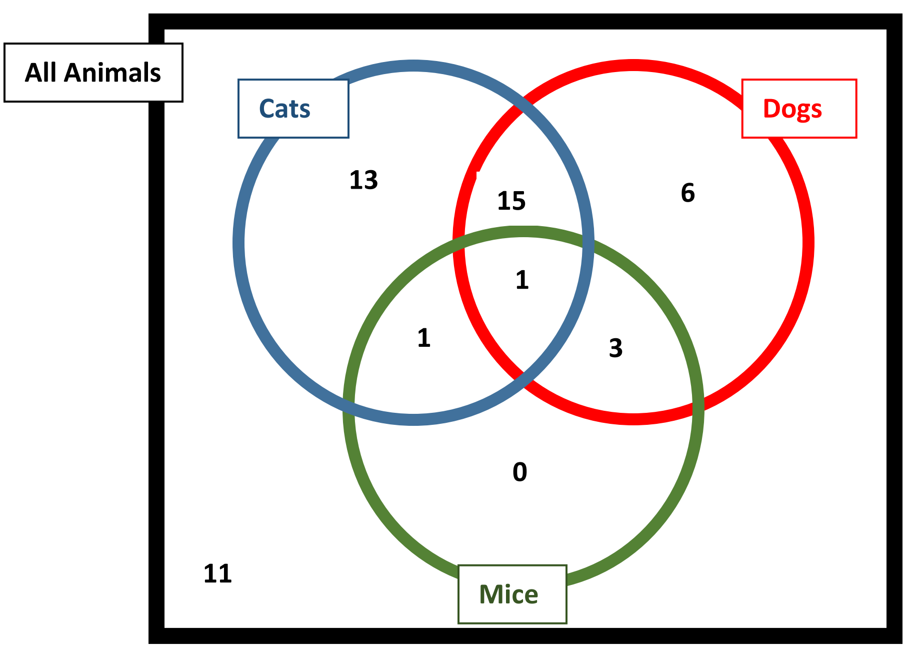
The students in Mr. Albert's math class conducted a survey of 50 students about their pets. Below are the results:

- 30 students have cats
- 25 students have dogs
- 16 students have both dogs and cats
- 5 students have mice
- 4 students have both dogs and mice
- 2 students have both cats and mice
- 1 student has all three

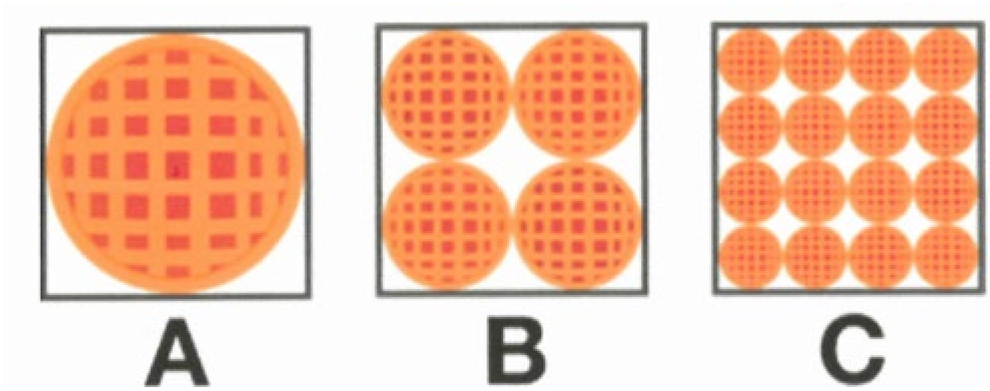
Your task is to determine how many of the 50 surveyed students have no pet?

11

Solution



How Much Pie?



All squares are the same size, and the pies within each square are identical.

The Tau Bakery packs its cherry pies in three different packages. You love cherry pie and want to figure out which package gives you the most pie. Decide whether you want to select Package A, Package B, or Package C. You can assume that all pies the Tau Bakery makes are baked in two-inch deep pie plates.

Your task is to determine which package contains the most pie.

Solution: They are all the same.

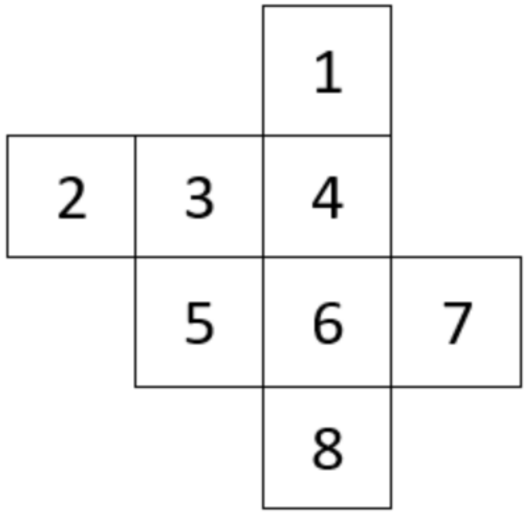
Box A: $A = \pi r^2$ and with just one pie that is it.

Box B: One pie is $A = \pi(\frac{r}{2})^2$=$1/4\pi r^2$ and with 4 pies $A = 4(\frac{1}{4})\pi r^2 = \pi r^2$

Box C: One pie is $A = \pi(\frac{r}{4})^2$=$1/16\pi r^2$ and with 16 pies $A = 16(\frac{1}{16})\pi r^2 = \pi r^2$

Mathematical Constants

Eight symbol cards lie face down on a table in the positions shown in the diagram below.

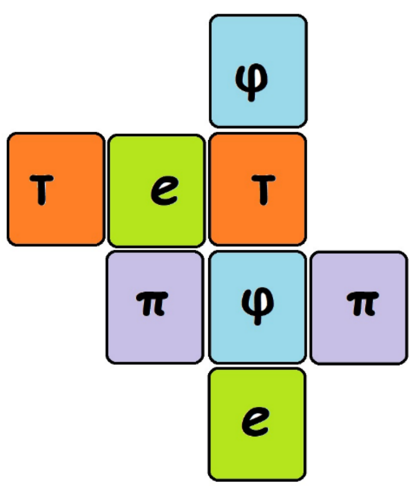


Of the eight cards:

- Every pi (π) borders a phi (φ).
- Every phi (φ) borders a tau (τ).
- Every tau (τ) borders an Euler's number (e).
- No tau (τ) borders a pi (π).
- No two cards of the same kind border each other.
- There are two pi (π), two phi (φ), two tau (τ), and two Euler's number (e).

Your task is to determine the position of all the cards. If you are correct on your first attempt, your team will be awarded four points. If you are incorrect on the first attempt, the master teacher will give you a hint and deduct one point from the points possible for this problem.

Solution: The hint is φ is in the number 6 position.



Maximum Slide

A company wants to design a slide with maximum steepness. The slide will be built by attaching panels to a large steel frame arranged as a coordinate grid. One point of the slide must pass through the point (1, 2). A second point of the slide must pass through an ordered pair of the form (*x, y*) where the values of *x* and *y* are integers 1 through 9. Each integer can be used, at most, one time in creating the ordered pairs.

Your task is to answer the following questions:

- What ordered pair will create the slide with maximum steepness? (You can't slide down a vertical panel—you'll just fall—so an undefined steepness is not allowed.)
- If the slide is expanded, what *y*-value would correspond to *x* = 20?
- What is the equation of the line that represents the slide?

Solution:

The ordered pair is (2, 9)

When x=20, y=135

Equation is y = 7x - 5

No Pi Here

Professor Archimedes challenged his students to find numbers composed of three different digits such that each is divisible by the square of the sum of those digits. He gave them an example to start their thinking.

Example: In the case of 112, the digits sum is 4, the square of which is 16, and 112 can be divided by 16 without a remainder, but unfortunately 112 does not contain three **DIFFERENT** digits.

There is more than one number that will fit these conditions. Your team will earn 3 points for one solution. It is possible to earn 2 more points for a second solution and 1 additional point for a third solution.

Your task is to determine at least one, but no more than three, solution(s) that fit the challenge that Professor Archimedes gave his students.

Solution: The numbers are 162, 243, 324, 392, 405, 512, 605, 648, 810, and 972.

Parallel Lines and Transversal

Using the digits 1 to 9 at most one time each, fill in the boxes so that two of the lines are parallel and the third line is a transversal that is as close to perpendicular to the parallel lines as possible. Use the provided digit cards to help you solve the equations.

$$\square x + \square y = \square$$

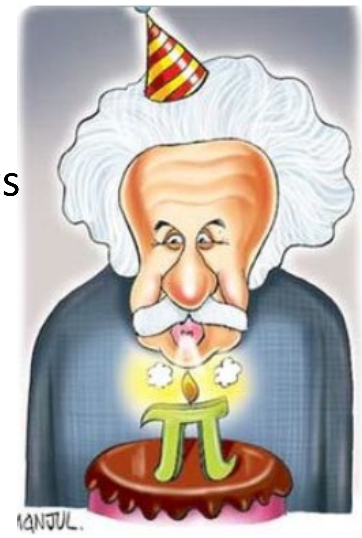
$$\square x + \square y = \square$$

$$\square x + \square y = \square$$

Your task is to complete the above equations.

Since the slope of a perpendicular line should be the negative reciprocal of the other line, and since we can't make it negative, then the slope that is closest to zero should be the closest to perpendicular. Accordingly, the best answer so far is 6x + 3y = 7, 4x + 2y = 8, x + 9y = 5

Supplementary Complement



On his birthday, Albert Einstein was pondering angles. He observed an angle that measured at least $\frac{2}{3}$ of its supplement and, at most, 5 times its complement.

Your task is to determine the difference between the largest and smallest possible degree measures of the angle.

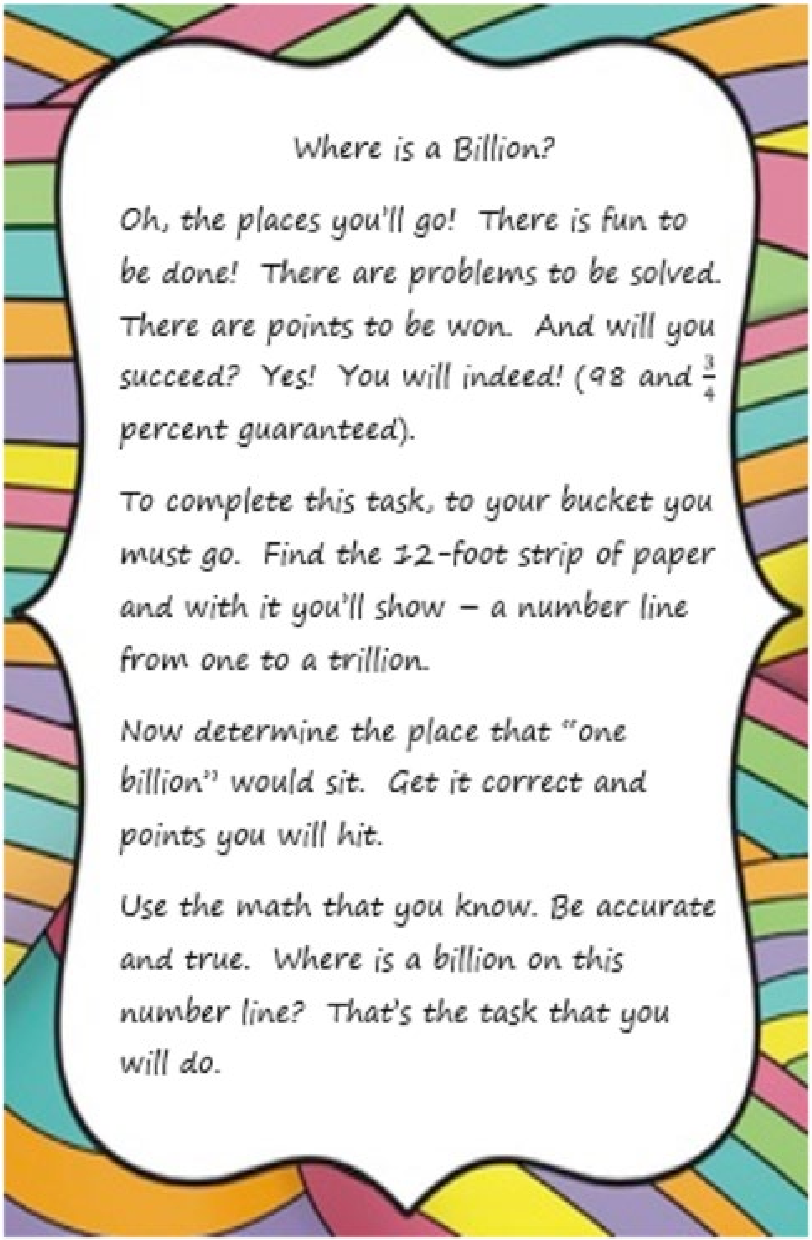
Be prepared to justify your solution.

Solution: 3

Angle ≥ 2/3(180-Angle)	Angle ≤ 5(90-Angle)
A ≥ 120 -2/3A	A ≤ 450 – 5A
5/3A ≥ 120	6A ≤ 450
A ≥ 72	A ≤ 75

Difference: 78 – 75 = 3

Where is a Billion?



Where is a Billion?

Oh, the places you'll go! There is fun to be done! There are problems to be solved. There are points to be won. And will you succeed? Yes! You will indeed! (98 and $\frac{3}{4}$ percent guaranteed).

To complete this task, to your bucket you must go. Find the 12-foot strip of paper and with it you'll show – a number line from one to a trillion.

Now determine the place that "one billion" would sit. Get it correct and points you will hit.

Use the math that you know. Be accurate and true. Where is a billion on this number line? That's the task that you will do.

Task:

- Use the adding machine tape as a number line.
- Place the end points, "zero" and "one trillion" on the number line.
- Mark one billion on the number line.
- Bring the number line to the Master Teacher.
- Be ready to explain the math you used to justify the placement of one billion.

Solution:

There are 1,000 billions in a trillion.

12' = 144 in

144 ÷ 1,000 = 0.144 inch is where you would find 1 billion.

Placement should be around $\frac{1}{8}$ of an inch.

Wipeout Answers

a) 3
b) 4
c) 12
d) $N = 13$, # removed = 9
e) $N = 51$, # removed = 38

Extension: