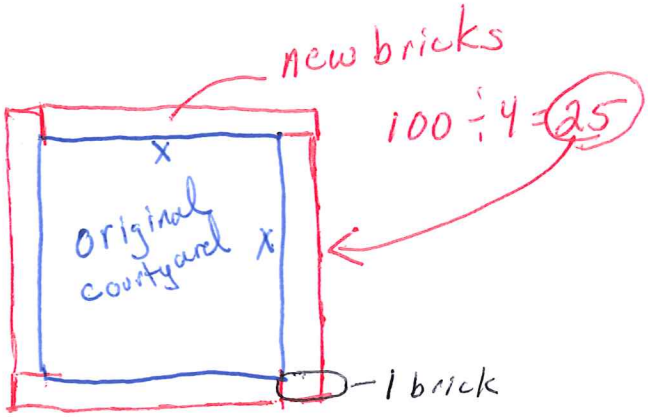
And 100 More

Liam the Mischievous Leprechaun

A man completely paved a square courtyard with square tiles. Liam arrived one night and decided the courtyard was too small. He dug up the courtyard, removed all the tiles, stacked them neatly, and added 100 more to the stack. The next morning the man returned to the courtyard to find a stack of tiles with a note instructing him to build another square courtyard using all of the tiles in the stack.



$x = 25 - 1 = 24$

new dimension = 24 + 2 = 26

or 25 + 1 = 26

Your task is to determine how many tiles were used in the second courtyard. All the tiles were the same size. Be prepared to justify your solution.

New courtyard $26 \times 26 = 676$

original courtyard $24 \times 24 = 576$

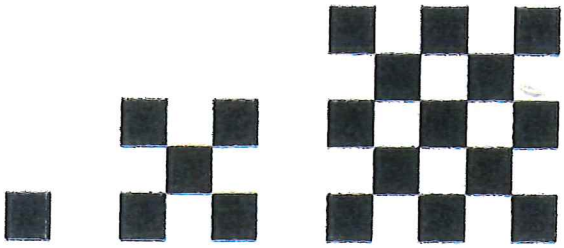
When you have solved this problem, go to Room 1069 to present your solution to the Master Teacher.

$676 - 576 = 100$

Checkerboard Pattern

Your task is to determine an explicit formula for the number of tiles in the n^{th} step for the following pattern.

The tiles are just the black squares.



1st 2nd 3rd

1 tile 5 tiles 13 tiles

n = the step #

T = # of tiles

4th 25 tiles

Be prepared to justify each aspect of your formula with respect to the pictorial representation.

Before going to the Master Teacher, decide whether you are trying for:

1 Puzzle Piece (formula without pictorial representation)

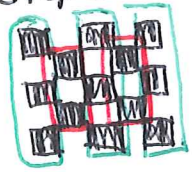
Or $T = 2n^2 - 2n + 1$

4 Puzzle Pieces (formula with complete pictorial representation)

Once you report to the Master Teacher, you are locked into your decision.

This is one way

step 3



$3 \times 3 \rightarrow n \times n = n^2$

$2 \times 2 \rightarrow (n-1)(n-1) = n^2 - 2n + 1$

add together

$n^2 + n^2 - 2n + 1$

$2n^2 - 2n + 1$

When you have solved this problem, go to Room 1021 to present your solution to the Master Teacher.

Coin Game

Sparkles O'Looney and Blarney O'Stone each has a set of gold coins. They decided to play four rounds of a game where, after each round, the loser must give the winner as many gold coins as the winner has at that time. Both leprechauns ended up with 16 gold coins. If Sparkles won the first two rounds and Blarney won the last two rounds, how many gold coins did each one have at the start of the game?

Your task is to determine how many gold coins Sparkles had and how many gold coins Blarney had when the game began.

When you have solved this task, go to Room 1065 to present your solution to the Master Teacher.

One Way: Work Backwards

	After round 4	After round 3	After round 2	After round 1	start
Sparkles	16	24	28	14	7
Blarney	16	8	4	18	25

Growing Squares

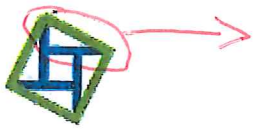
Molly McDoodle was doodling the other day and drew a little square like this:

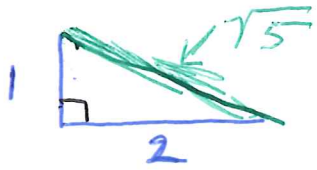
Area = 1 sq unit Perimeter = 4 units

And she supposed that the side was one (something) long. Well, she wondered what would happen if she drew the four lines a bit longer, in fact twice as long so that the extra bits stuck out.

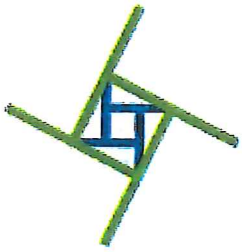
This was quite a nice little design, she thought, and then she noticed that it looked as though the ends of these lines could make a square. So, she drew one! She used a different color to show this new square.

Use Pythagorean Theorem

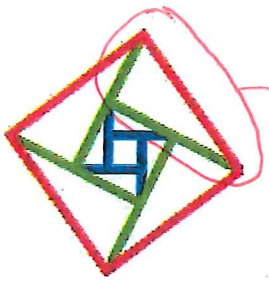


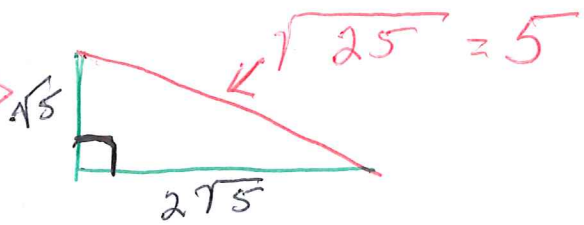


Now, mathematical patterns usually go on repeating themselves so she used that idea to pretend that this new green square was her first one and so she drew the extra bits again, so that the lines were twice as long as the square. Her new shape looked like this:



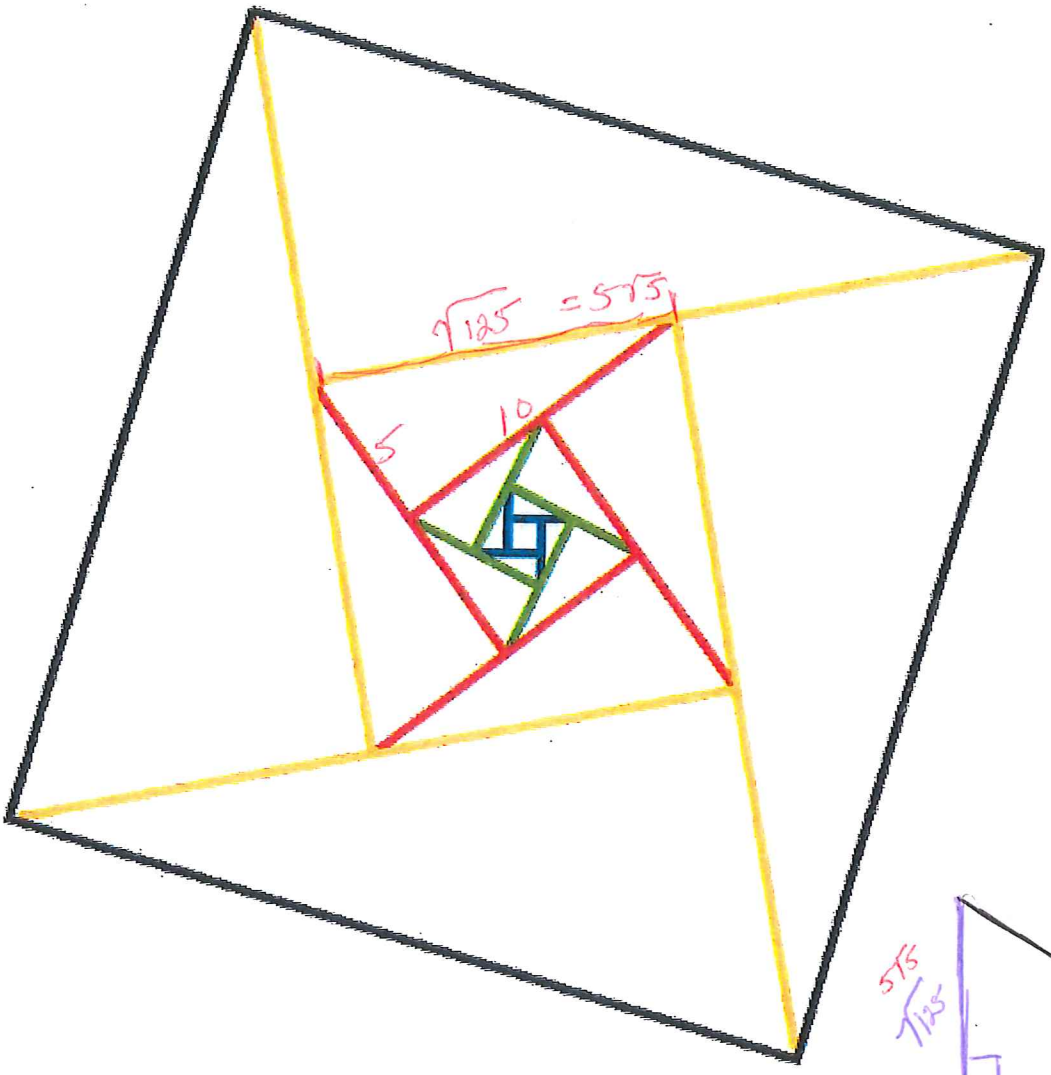
So she went on! A new square appeared, now red!

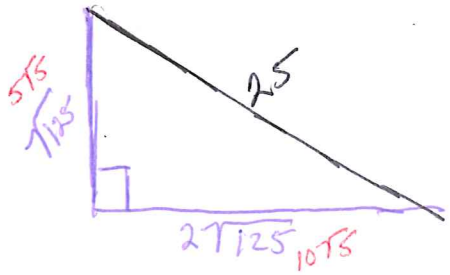




(continued on back)

She really liked what was happening here. She continued with this mathematical pattern until creating this final drawing.





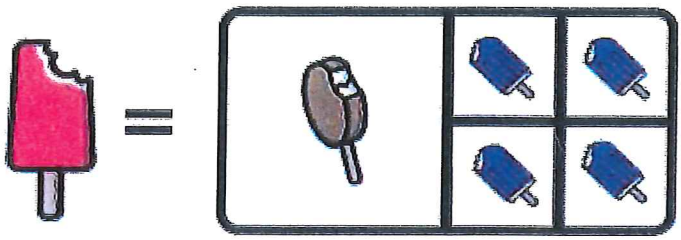
Your task is to determine the perimeter and area of the black square if the area of the blue square is one square unit.

$A = 25^2 = 625$ sq units $\qquad P = 100$ units

When you have solved this task, go to Room 1016 to present your solution to the Master Teacher.

Ice Cream Challenge

Your task is to determine the value of each frozen treat.
Be prepared to justify your solution.

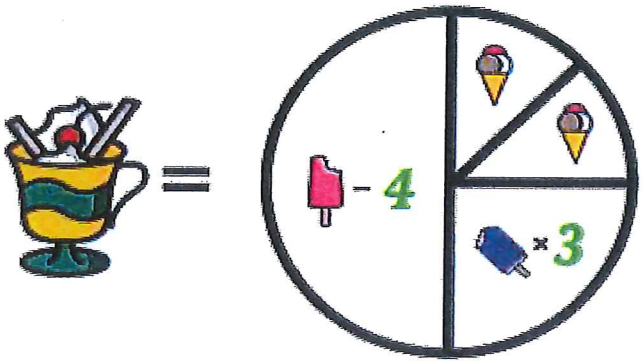


Area Model

$4B = 1E$

$2E = 1R$

$8B = 1R$



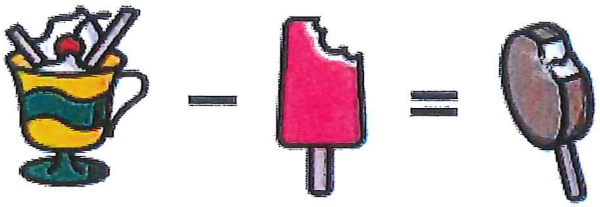
$2C = 3B$

$2C + 3B = R - 4$

$6B = R - 4$

$P = 6B + (8B - 4)$

$P = 14B - 4$



$(14B - 4) - (8B) = 4B$

$6B - 4 = 4B$

$2B = 4$

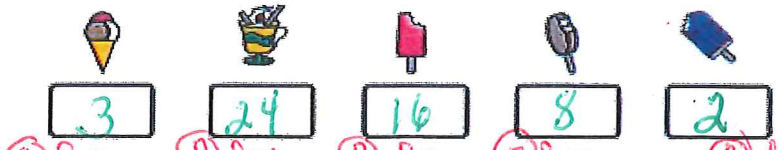
$\boxed{B = 2}$

$E + C = 11$

$4B + C = 11$

$4(2) + C = 11$

$\boxed{C = 3}$

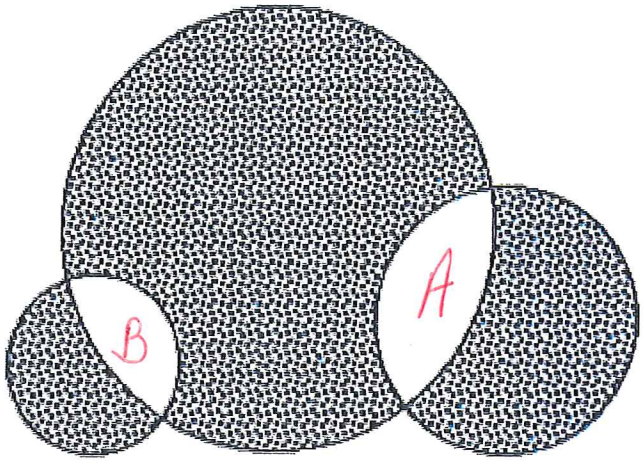


(C) for Cone (P) for Parfait (R) for Red popsicle (E) for Eskimopie (B) for Blue popsicle

When you have solved this problem, go to Room 1019 to present your solution to the Master Teacher.

There are many ways to explain this.

Inside And Out



The circles above have radii 12 units, 8 units and 5 units.

Your task is to determine the difference between the shaded region inside the big circle and the shaded region outside the big circle.

Area of largest circle = 144π

Area of (mid size) circle = 64π

Area of smallest circle = 25π

marked above in Red

Area inside the largest circle = $144\pi - (A+B)$

Area outside the largest circle = $(64\pi - A) + (25\pi - B)$

Difference = $[144\pi - (A+B)] - [(64\pi - A) + (25\pi - B)]$

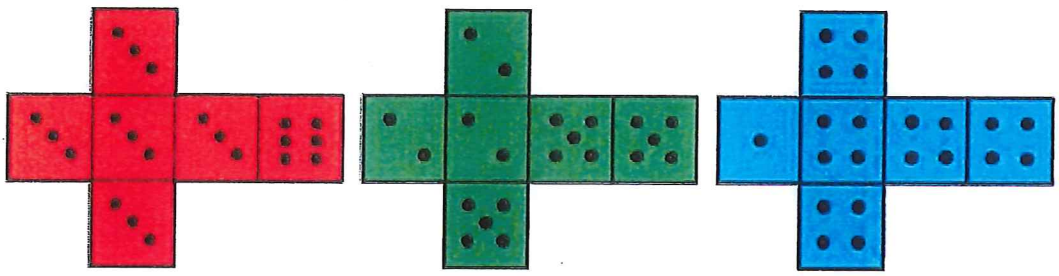
$144\pi - A - B - 64\pi + A - 25\pi + B = 55\pi$

When you have solved this problem, go to Room 1056 to present your solution to the Master Teacher.

Lucky Dice

Two people are playing a game using the special colored dice shown below. There is one red die, one green die, and one blue die. To play the game, each person picks one die.

- Each player rolls their die and the person who rolls the highest number wins the round.
- The players roll their dice 30 times and keep track of who wins each round.
- Whoever has won the most rounds after 30 rolls wins the game.



Your task is not to play the game, you should:

A. Pick any two colors and tell which color would most likely win and give the probability of that color winning.

P(Red winning over Green) = 21/36 P(Blue winning over Red) = 25/36 P(Green winning over Blue) = 21/36

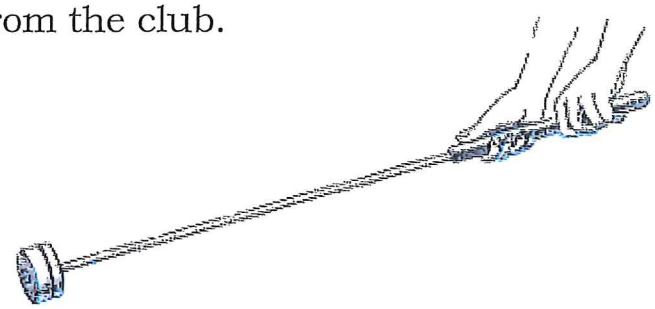
B. If you were playing the game, is it better to choose your color first or second? Explain. Go second, no matter what color your opponent chooses there is always a die on the table that has a better chance of winning which you can choose.

When you have solved this problem, go to Room 1010 to present your solution to the Master Teacher.

Adapted from Illustrative Mathematics

Mulligan's Club

Mulligan's Company wants to manufacture a practice golf club that will have a variable amount of weight at the end of the club. The golfer will begin by placing a1-ounce weight on the end of the club and then build up gradually in 1-ounce steps to reach a maximum of 15 ounces. Weights can be added and removed from the club.



Séamus Power suggests that it will be most convenient to have as small a number of individual weights as possible. Mulligan's Company likes this idea, but they want to know the specific value of each of the weights Séamus Power suggests.

Your task is to determine the smallest number of individual weights and their values that Séamus can have available for a session with his practice club to have any weight from one to fifteen ounces in one-ounce increments. (Ignore the weight of the club itself.)

Make the weights in powers-of-2

1 oz, 2 oz, 4 oz, 8 oz. [4 weights]

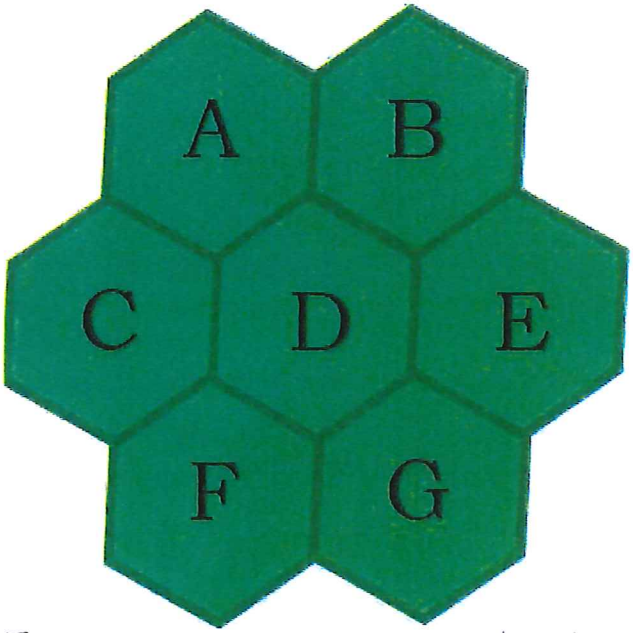
1 → 1 oz
2 → ~~2~~ 2 oz
3 → 2+1
4 → 4
5 → 4+1
6 → 4+2
7 → 4+2+1
8 → 8
9 → 8+1
10 → 8+2
11 → 8+2+1
12 → 8+4
13 → 8+4+1
14 → 8+4+2
15 → 8+4+2+1

When you have solved this task, go to Room 1013 to present your solution to the Master Teacher.

Patchy Situation

The Leprechaun Quilting Society is preparing a patchwork quilt to be auctioned off to raise money for the Math League. Just as the members themselves are diverse, the quilt is a colorful medley of fabrics and patterns. A section of the finished quilt is shown below. Each piece is made of a different material in a different pattern and in a different basic color. From the information given, determine, for each piece (A through G in the illustration), its basic color, pattern (one pattern is pentagrams), and fabric (one section is made out of wool).

1. The zigzag patch is predominantly yellow. It is adjacent to patch D, the pink patch, and the green dotted patch. Patch G is satin.
2. The navy patch is adjacent to the geometric patch (which is not the polyester one), the one with a π pattern, and patch D.
3. The parallel line patch (which isn't patch E) is in a direct line with the brown patch, with only patch D between them. The brown patch is adjacent to the nylon patch.
4. Patch A is red, but it isn't the velvet one with the π pattern. Patch C is checked, but isn't the pink, silk one.
5. The orange patch is made of cotton.



Your task is to the color, design, and fabric for each of the hexagonal quilt pieces. List these by the hexagonal quilt pieces' letters.

A - red, geometric, wool

B - pink, parallel lines, silk

C - navy, checked, nylon

D - orange, pentagrams, cotton

E - yellow, zigzag, polyester

F - Brown, π, velvet

G - green, dotted, satin

When you have solved this task, go to Room 1009 to present your solution to the Master Teacher.

Relationships

Sean and Conner, the playful Leprechauns, have found a box of golden Relational GeoSolids and a bag of rice. Since they are always drawn to gold, they started playing with them. They discovered that when they filled them, there were relationships between the solids. In their wonderful math class, taught by Lady MacMath, they had learned a formula for the area of a circle. They noticed that some of the shapes had involved circles. From this, they developed a way to find the volume of a cylinder.

Given
Area of circle is πr^2

Volume of cylinder is a circle base times height (In relational solids $h = 2r$)

$V = \pi r^2 h$

$V = \pi r^2 \cdot 2r$

$V = 2\pi r^3$

The overachieving leprechauns continued playing with the solids and found more relationships until they developed a formula for finding the volume of the sphere. $V = \frac{4}{3}\pi r^3$

Your task is to be as clever as the leprechauns and, using the relationships, you notice in the GeoSolids, demonstrate to the Master Teacher how to find the volume of a sphere, and state the formula. The Master Teacher will expect to see your algebraic reasoning.

Through investigation with rice & solids

3 cones = 1 cylinder

or

1 cone = $\frac{1}{3}$ cylinder

and 1 cone = 1 hemisphere

1 sphere = 2 hemispheres

When you have solved this task, go to Room 1062 to present your solution to the Master Teacher.

Volume 1 cylinder = $2\pi r^3$

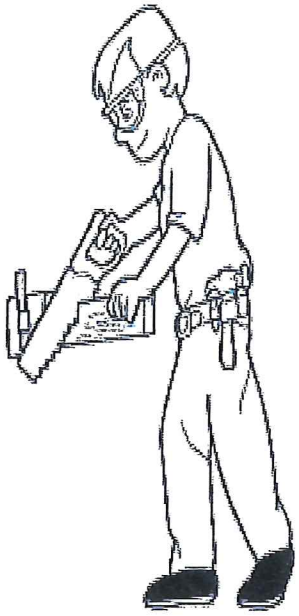
So Volume 1 cone = $\frac{1}{3}(2\pi r^3)$

1 sphere = 2 cones

Volume 1 sphere = $2\left(\frac{1}{3}\right)(2\pi r^3)$

Volume 1 sphere = $\left(\frac{4}{3}\right)\pi r^3$

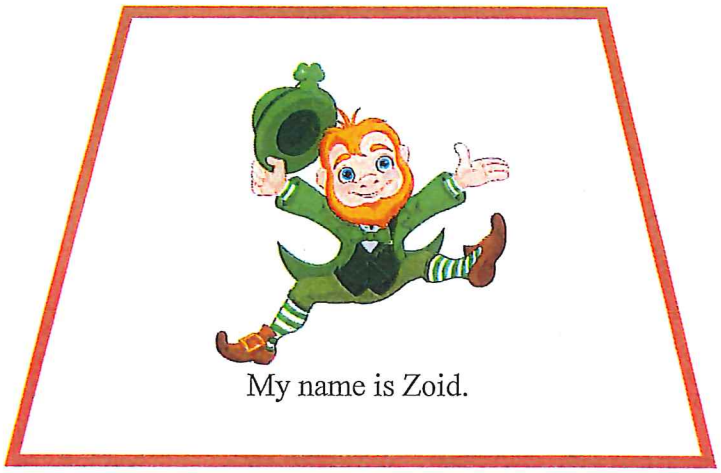
The Long and Short of It



Ryan O'Toole is building a trapezoidal leprechaun trap. He has a wooden board measuring (y + 8) feet in length to create the bases for the leprechaun trap. The board is cut into two pieces in the ratio 3:7.

short base

long base



set up ratio

$$\frac{\text{short base}}{\text{whole board}}$$

$$\frac{3}{10} = \frac{\text{short base}}{y+8}$$

$$10(\text{short base}) = 3(y+8)$$

$$\text{short base} = \frac{3(y+8)}{10}$$

Your task is to determine the length of the board for the shortest base.

When you have solved this task, go to Room 1070 to present your solution to the Master Teacher.